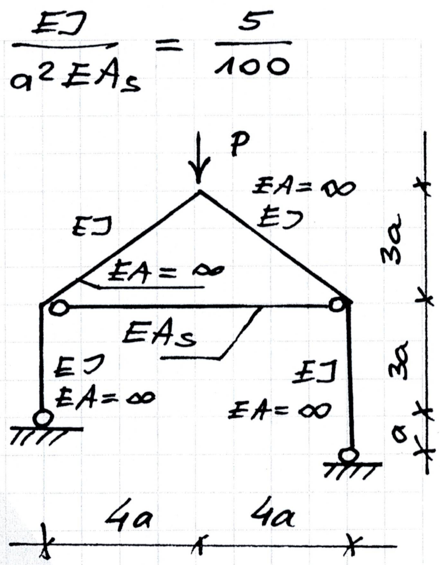
Egzamin pisemny z Mechaniki Konstrukcji I, 4 IX 2023 r.

NAZWISKO imię				
Grupa	Data zaliczenia ćwiczeń		Numer albumu	
Ocena zadania 1	Ocena zadania 2	Ocena zadania 3	Ocena z egzaminu (po egz. ustnym)	Ocena łączna Data

Zadanie 1

Dana jest rama płaska obciążona jak na rysunku. Sporządzić wykres momentów zginających metodą sił.

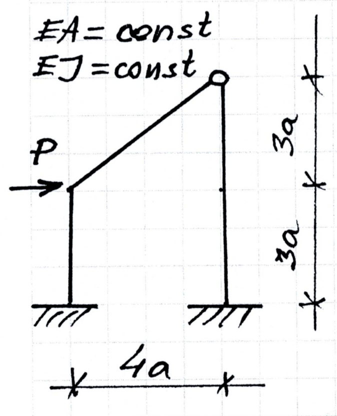
(Problem1.*Given is the frame loaded as shown in the figure. Construct the diagram of the bending moments)*



Zadanie 2

Dana jest rama płaska jak na rysunku. Utworzyć układ równań macierzowej metody przemieszczeń.

(Problem 2. *Given is the plane frame as in the figure; write down the equations of the displacement method in the matrix form)*



Zadanie 3

Dany jest pręt prosty pryzmatyczny.
Wyprowadzić równanie transformacyjne schematu z dwoma utwierdzeniami:

$$(1)\,\Phi_i = \frac{2EJ}{l}\left(2\varphi_i + \varphi_k - 3\psi\right) + \Phi_i^o$$

i wykazać następnie, że ten związek spełnia kryterium ruchu sztywnego.

(Problem 3. *Given is a straight prismatic bar. Derive the slope-deflection equation (1) for a scheme with two clamped ends and then prove that this formula satisfies the rigid body criterion.)*

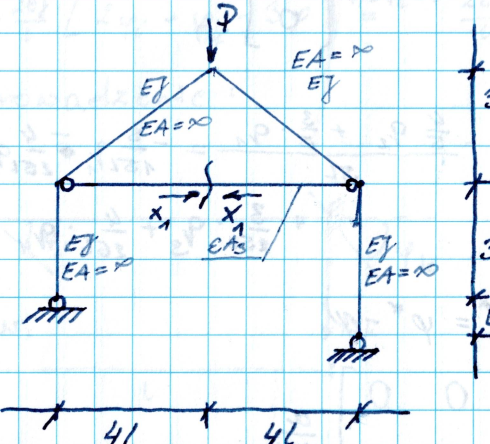
Egzamin pisemny

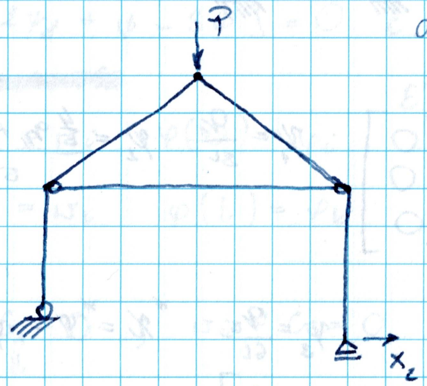
Mechanika Konstrukcji

04.09.2023 r.

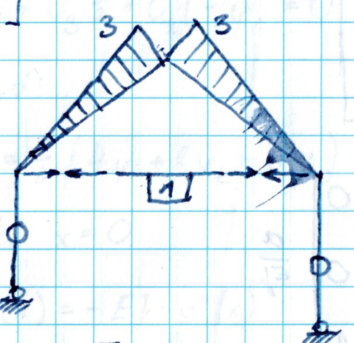
Zadanie 1.

$$\frac{EJ}{a^2 EA_s} = \frac{5}{100}$$

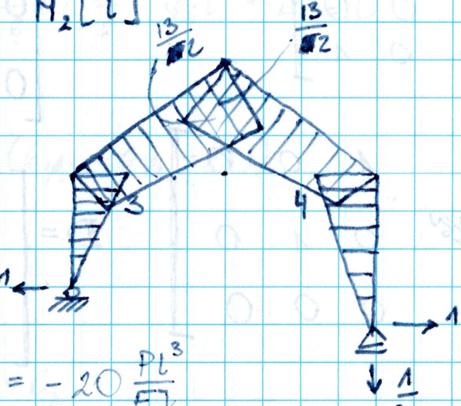




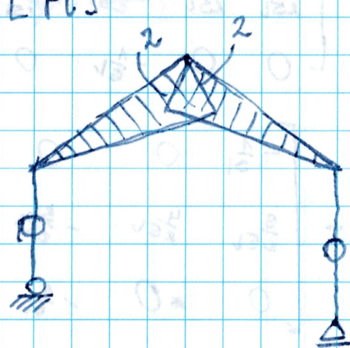
$M_1\,[l]$



$M_2\,[l]$



$M_0\,[Pl]$



$$\delta_{10} = -\frac{1}{EJ}\left[2\cdot\left(\frac{1}{2}\cdot 2Pl\cdot 5l\cdot\frac{2}{3}\cdot 3l\right)\right] = -20\,\frac{Pl^3}{EJ}$$

$$\delta_{20} = \frac{1}{EJ}\left[\frac{1}{2}\cdot 2Pl\cdot 5l\cdot\left(\frac{1}{3}\cdot 3l + \frac{2}{3}\cdot\frac{13}{2}l\right) + \frac{1}{2}\cdot 2Pl\cdot 5l\cdot\left(\frac{1}{3}\cdot 4l + \frac{2}{3}\cdot\frac{13}{2}l\right)\right] = 55\,\frac{Pl^3}{EJ}$$

$$\delta_{11} = \frac{1}{EJ}\left[2\cdot\left(\frac{1}{2}\cdot 3l\cdot 5l\cdot\frac{2}{3}\cdot 3l\right)\right] + \frac{1}{EA}\left[1\cdot 1\cdot 8l\right] = 30\,\frac{l^3}{EJ} + \frac{8}{20}\frac{l^3}{EJ} = \frac{152}{5}\frac{l^3}{EJ}$$

$$\delta_{12} = \delta_{21} = -\frac{1}{EJ}\left[\frac{1}{2}\cdot 3l\cdot 5l\cdot\left(\frac{1}{3}\cdot 3l + \frac{2}{3}\cdot\frac{13}{2}l\right) + \frac{1}{2}\cdot 3l\cdot 5l\cdot\left(\frac{1}{3}\cdot 4l + \frac{2}{3}\cdot\frac{13}{2}l\right)\right] = -\frac{165}{2}\frac{l^3}{EJ}$$

$$\delta_{22} = \frac{1}{EJ}\Big[\frac{1}{2}\cdot 3l\cdot 3l\cdot\frac{1}{3}\cdot 3l + \frac{1}{2}\cdot 3l\cdot 5l\cdot\left(\frac{2}{3}\cdot 3l + \frac{1}{3}\cdot\frac{13}{2}l\right) + \frac{1}{2}\cdot\frac{13}{2}l\cdot 5l\cdot\left(\frac{1}{3}\cdot 3l + \frac{2}{3}\cdot\frac{13}{2}l\right) +$$

$$+ \frac{1}{2}\cdot\frac{13}{2}l\cdot 5l\cdot\left(\frac{2}{3}\cdot\frac{13}{2}l + \frac{1}{3}\cdot 4l\right) + \frac{1}{2}\cdot 4l\cdot 5l\cdot\left(\frac{2}{3}\cdot 4l + \frac{1}{3}\cdot\frac{13}{2}l\right) + \frac{1}{2}\cdot 4l\cdot 4l\cdot\frac{2}{3}\cdot 4l\Big] =$$

$$= \frac{866}{3}\frac{l^3}{EJ}$$

$$\mathbb{D} = \begin{bmatrix}\delta_{11} & \delta_{12}\\ \delta_{12} & \delta_{22}\end{bmatrix} \qquad d_0 = \begin{bmatrix}\delta_{10}\\ \delta_{20}\end{bmatrix}$$

$$\mathbb{D}X + d_0 = 0$$

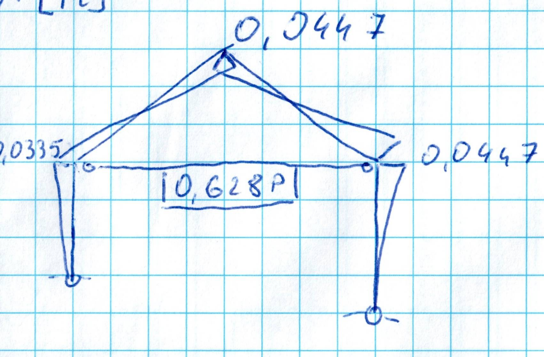
$$\frac{152}{5}\frac{l^3}{EJ}\cdot X_1 - \frac{165}{2}\frac{l^3}{EJ}\cdot X_2 = 20\frac{Pl^3}{EJ}$$

$$-\frac{165}{2}\frac{l^3}{EJ}\cdot X_1 + \frac{866}{3}\frac{l^3}{EJ}\cdot X_2 = -55\frac{Pl^3}{EJ}$$

$$X_1 = 0{,}628\,P$$

$$X_2 = \ominus 0{,}0112\,P$$

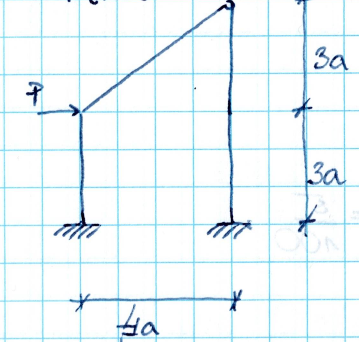
$M\,[Pl]$

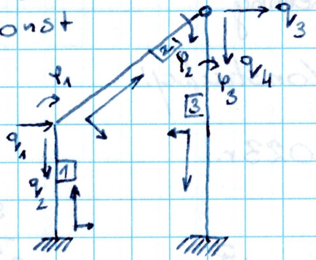


Zadanie 2.

$EA = const$

$EJ = const$





$$\mathbb{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix} \qquad \mathbb{Q} = \begin{bmatrix} P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbb{B} = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{4}{5} & \frac{3}{5} & \frac{4}{5} & -\frac{3}{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$\psi_1 = \frac{q_1}{3l} \qquad \psi_2 = \frac{\frac{4}{5} q_4 + \frac{3}{5} q_3}{5l} - \frac{\frac{4}{5} q_2 + \frac{3}{5} \cdot q_1}{5l} = -\frac{3}{15l} q_1 - \frac{4}{25l} q_2 + \frac{3}{25l} \cdot q_3 + \frac{4}{25l} \cdot q_4$$

$$\psi_3 = \frac{q_3}{6l} \qquad {}^*\chi = {}^*\varphi - \psi \qquad \chi^* = \varphi^* - \psi$$

$${}^*\mathbb{B} = \begin{bmatrix} -\frac{1}{3l} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{25l} & \frac{4}{25l} & -\frac{3}{25l} & -\frac{4}{25l} & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{6l} & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbb{E} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{5} & 0 \\ 0 & 0 & \frac{1}{6} \end{bmatrix} \frac{a}{EA}$$

$$\mathbb{B}^* = \begin{bmatrix} -\frac{1}{3l} & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{3}{25l} & \frac{4}{25l} & -\frac{3}{25l} & -\frac{4}{25l} & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{6l} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbb{D} = \begin{bmatrix} \frac{2}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \frac{a}{EJ}$$

$$\mathbb{K} = \mathbb{B}^T \mathbb{E} \mathbb{B} + 2\,{}^*\mathbb{B}^T \mathbb{D}\,{}^*\mathbb{B} + {}^*\mathbb{B}^T \mathbb{D} \mathbb{B}^T + \mathbb{B}^{*T} \mathbb{D}\,{}^*\mathbb{B} + 2\mathbb{B}^{*T} \mathbb{D} \mathbb{B}^*$$

$$\mathbb{q} = \mathbb{K}^{-1} \cdot \mathbb{Q}$$

$$\mathbb{N} = \mathbb{E} \mathbb{B} \mathbb{q}$$

$${}^*\Phi = \mathbb{D} \left(2\,{}^*\mathbb{B} \mathbb{q} + \mathbb{B}^* \cdot \mathbb{q} \right)$$

$$\Phi^* = \mathbb{D} \left({}^*\mathbb{B} \mathbb{q} + 2\mathbb{B}^* \mathbb{q} \right)$$

Zadanie 3.

Ruch sztywny: $w_k = w_i + \psi \cdot l$, wtedy $\varphi_i = \varphi_k = \psi$

Kryterium ruchu sztywnego mówi, że sił wewnętrzne się na nim zerują, więc:

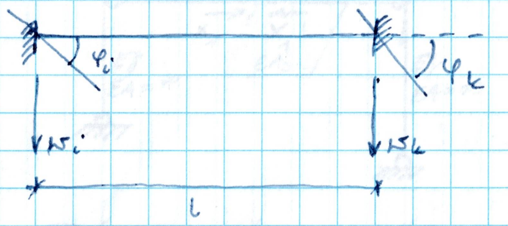
$\Phi_i = \frac{2EJ}{l}\left[2\varphi_i + \varphi_k - 3\psi\right] \overset{\varphi_i=\varphi_k=\psi}{=} \frac{2EJ}{l}\cdot\left[2\psi + \psi - 3\psi\right] = 0$ c. k. d.

Wyprowadzenie:

$EJ\, w^{IV}(x) = 0$

$w(0) = w_i \quad \varphi(0) = \varphi_i$

$w(l) = w_k \quad \varphi(l) = \varphi_k$



$\varphi_i + C_1 = 0 \quad ; \quad w_i + C_0 = 0$

$\Downarrow$

$C_1 = -\varphi_i \quad ; \quad C_0 = -w_i$

$\varphi_k + C_3\frac{l^2}{2} + C_2 l - \varphi_i = 0 \quad ; \quad w_k + C_3\frac{l^3}{6} + C_2\frac{l^2}{2} - \varphi_i l + w_i = 0$ oraz $\psi = \frac{w_k - w_i}{l}$

Stąd:

$C_2 = \frac{1}{l}(4\varphi_i + 2\varphi_k - 6\psi) \qquad C_3 = \frac{1}{l^2}(6\varphi_i + 6\varphi_k - 12\psi)$

Dla $x = 0$

$M(x) = -EJ\cdot w''(x)$

$w''(0) + C_3 \cdot 0 + C_2 = 0 \Rightarrow w''(0) = -C_2$

$\Phi_i = M(0) = -EJ(-C_2) = \frac{EJ}{l}(4\varphi_i + 2\varphi_k - 6\psi) = \frac{2EJ}{l}\cdot(2\varphi_i + \varphi_k - 3\psi)$