

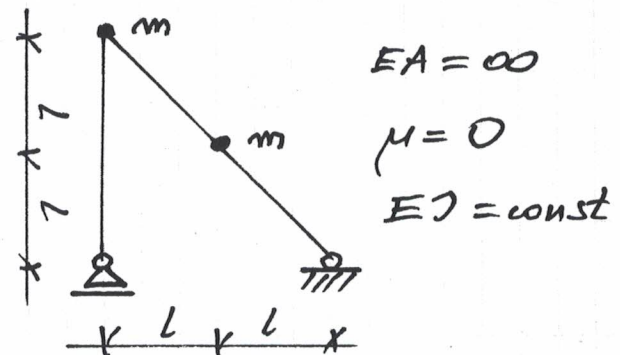
Imię i NAZWISKO				
Prowadzący ćwiczenia, nr grupy				
ocena zadania 1	ocena zadania 2	ocena zadania 3	ocena egz. pis.	Ocena ostateczna z egzaminu
				Ocena łączna

Zadanie 1

Dana jest rama z prętów nieważkich z masami skupionymi. Obliczyć częstości drgań własnych.

(Given is a frame of weightless bars with concentrated masses.

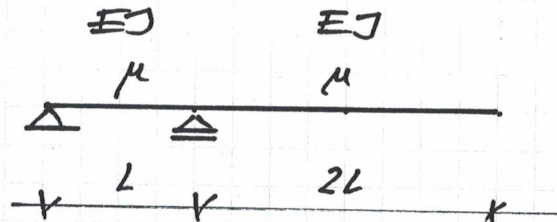
Compute the eigenfrequencies.)



Zadanie 2

Obliczyć pierwszą częstość drgań własnych danej belki.

(Compute the first eigenfrequency of the given beam)

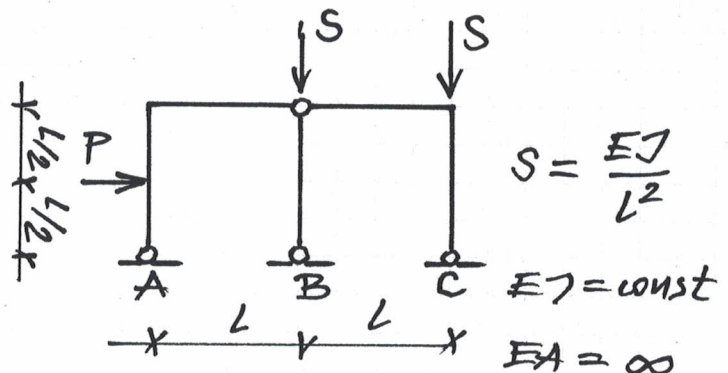


Zadanie 3

Dana jest rama płaska obciążona jak na rysunku.

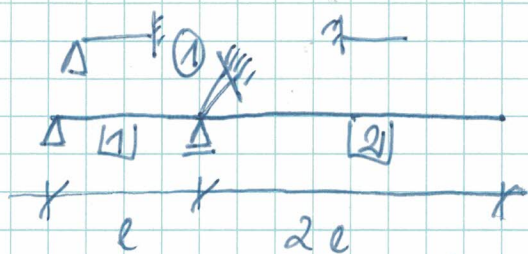
Zapisać równania określające wartość reakcji poziomej w C.

(Given is a plane frame loaded as in the figure. Write down the equations necessary to compute the horizontal reaction at C)



ZADANIE 2

UGW



$$q_1 = [\psi_1]$$

$$\lambda^{(1)} = l \sqrt[4]{\frac{\mu \omega^2}{EY}} = \lambda$$

$$\lambda^{(2)} = 2l \sqrt[4]{\frac{\mu \omega^2}{EY}} = 2\lambda$$

n.n. MP

$$\Phi_1^{(1)} + \Phi_1^{(2)} = 0$$

1) $\Delta \rightarrow$

$$\Phi_1^{(1)} = \frac{EY}{l} [\alpha'(x) \psi_1]$$

2) $\rightarrow$

$$\Phi_1^{(2)} = \frac{EY}{2l} [\alpha''(2x) \psi_1]$$

$$\frac{EY}{l} [\alpha'(x) + \frac{1}{2} \alpha''(2x)] \psi_1 = 0$$

$$f(x) = \alpha'(x) + \frac{1}{2} \alpha''(2x) = 0$$

$$x = 0,1$$

$$\alpha'(0,1) = 3,000$$

$$\alpha''(0,2) = -0,001$$

$$f(0,1) = 2,999$$

$$x = 0,7$$

$$\alpha'(0,7) = 2,995$$

$$\alpha''(1,4) = -1,841$$

$$f(0,7) = 2,075$$

$$x = 0,8$$

$$\alpha'(0,8) = 2,992$$

$$\alpha''(1,6) = -4,587$$

$$f(0,8) = 0,703$$

$$x = 0,9$$

$$\alpha'(0,9) = 2,987$$

$$\alpha''(1,8) = -22,622$$

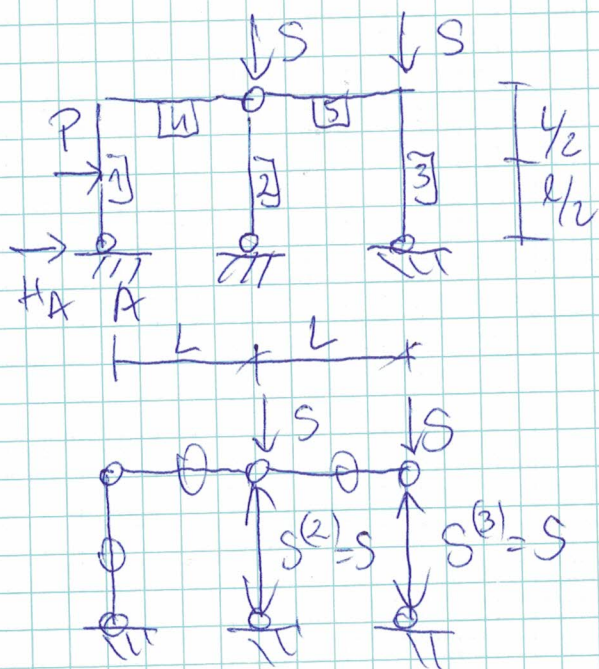
$$f(0,9) = -8,324$$

$$x_1 \approx 0,8$$

$$\omega_1 = \frac{x_1^2}{l^2} \frac{EY}{\mu} \approx 0,64 \frac{EY}{\mu l^2}$$

ZAD. 3.

EGZAMIN MK2 ST 8.08.2025r.

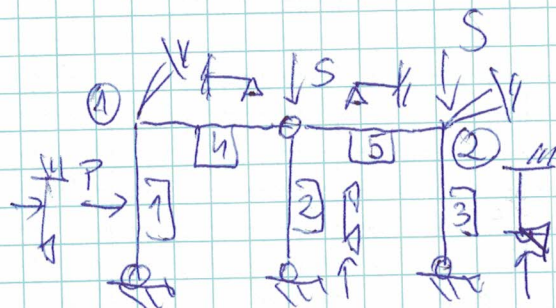


$$S = \frac{EJ}{L^2}$$

$$EJ = \text{const}$$

$$EA \rightarrow \infty$$

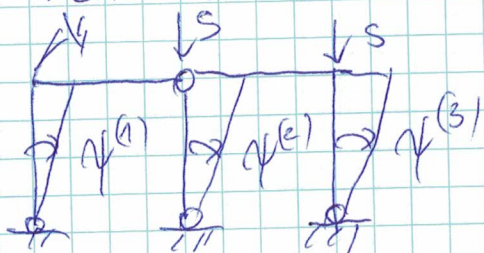
$$H_A = W_A^{(1)}$$



$$Q = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix}$$

$$KQ = Q$$

PLAN PRZEMIESŁCZEN



$$\psi^{(1)} = \psi^{(2)} = \psi^{(3)} = \psi$$

$$\bar{G} = L \sqrt{\frac{S}{EJ}} = L \sqrt{\frac{\frac{EJ}{L^2}}{EJ}} = 1$$

$l^{(1)} = L$	EJ	$S^{(1)} = 0$	$\bar{G}^{(1)} = 0 = 0$	$\psi^{(1)} = \psi$
$l^{(2)} = L$		$S^{(2)} = S$	$\bar{G}^{(2)} = \bar{G} = 1$	$\psi^{(2)} = \psi$
$l^{(3)} = L$		$S^{(3)} = S$	$\bar{G}^{(3)} = \bar{G} = 1$	$\psi^{(3)} = \psi$
$l^{(4)} = L$		$S^{(4)} = 0$	$\bar{G}^{(4)} = 0$	$\psi^{(4)} = 0$
$l^{(5)} = L$		$S^{(5)} = 0$	$\bar{G}^{(5)} = 0$	$\psi^{(5)} = 0$

RÓWNANIA

$$1) \sum M_1 = 0 \Rightarrow \phi_1^{(1)} + \phi_1^{(4)} = 0$$

$$2) \sum M_2 = 0 \Rightarrow \phi_2^{(3)} + \phi_2^{(5)} = 0$$

$$3) \phi_1^{(1)} \cdot \bar{\psi} + \phi_2^{(2)} \cdot \bar{\psi} + S \cdot L \cdot \psi \cdot \bar{\psi} + S \cdot L \cdot \psi \cdot \bar{\psi} + 4\bar{L}_2 = 0$$

$$\bar{L}_2 = P \cdot \frac{L}{2} \cdot \bar{\psi}$$

$$3) -\phi_1^{(1)} - \phi_2^{(3)} - 2S \cdot L \psi - \frac{PL}{2} = 0$$

$$\phi_1^{(1)} = \frac{3EJ}{L} (\psi_1 - \psi) + \frac{3}{16} PL$$

$$\phi_2^{(3)} = \frac{EJ}{L} [\alpha'(1) \cdot (\psi_2 - \psi)]$$

$$\phi_1^{(4)} = \frac{3EJ}{L} \psi_1$$

$$\phi_2^{(5)} = \frac{3EJ}{L} \psi_2$$

$$\frac{EJ}{L} \begin{bmatrix} 3+3 & \textcircled{2} & -3 \\ 0 & \alpha'(1)+3 & -\alpha'(1) \\ -3 & -\alpha'(1) & 3+\alpha'(1)+2 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi \end{bmatrix} = \begin{bmatrix} -\frac{3}{16} \\ \frac{3}{16} + \frac{1}{2} \end{bmatrix} PL$$

$$W_A^{(1)} = -\frac{EJ}{L^2} \left[\right.$$