EGZAMIN MK (MK3) KB studia mgr stacj. dnia 7 IX 2015 r.

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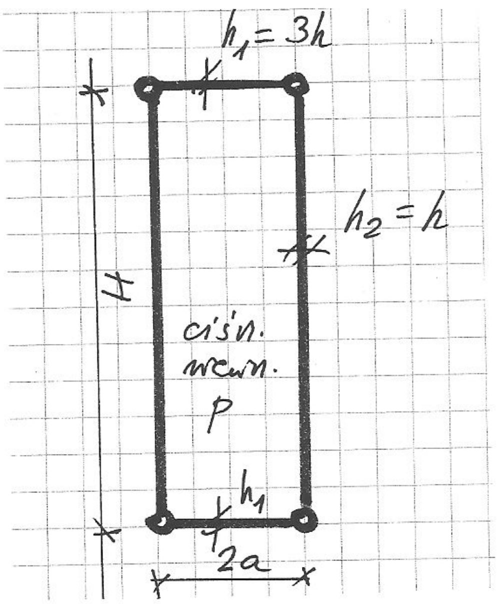
**Zadanie 1.**

Dany jest zbiornik walcowy zamknięty z obu stron płytami kołowymi. Przyjąć, że zbiornik jest długi.

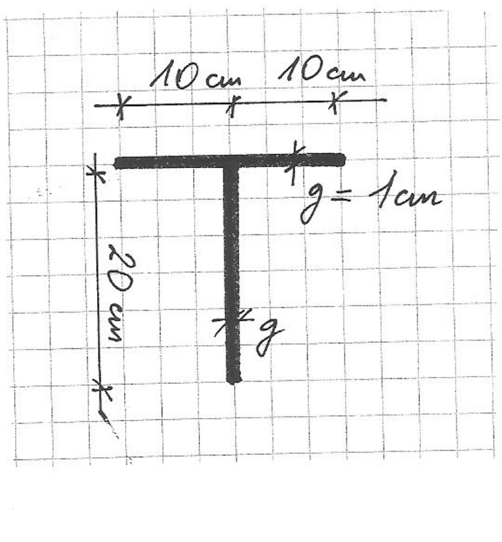
Znaleźć siły wewnętrzne $N_1, N_2, M_1, M_2$ w powłoce. Przyjąć E=30 GPa, wsp. Poissona=0.2.




$h = a/15$

**Zadanie 2.** Znaleźć siłę krytyczną P (przyłożoną w środku ciężkości SC przekroju) wyboczenia giętnoskrętnego pręta cienkościennego o danym przekroju, podpartego widełkowo, o długości $l = 3.0$ m.

Charakterystyki geometryczne wyznaczyć samodzielnie. Przyjąć E=205 GPa; wsp. Poissona=0.3.



**Exam on the Mechanics of Structures**
**7.09.2015**

PROBLEM #1
Consider a cylindrical shell closed by circular plates and loaded by an internal pressure $p$. Assume that the shell is long.

Calculate the internal forces $N_1, N_2$, $M_1, M_2$ in a cylinder.

Assume the Young modulus $E = 30GPa$ , Poisson ratio $\nu = 0.2$ .
See front page for shell dimensions.

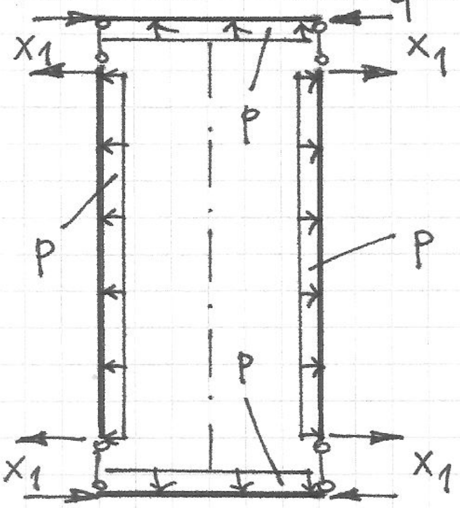
PROBLEM #2
Consider a thin-walled beam of length $l = 3.0m$, fork-supported and loaded by a force $P$ applied at cross-section's centroid $SC$ .

Calculate the value of the force $P$ critical for flexural-torsional buckling.

Assume the Young modulus $E = 30GPa$ and Poisson ratio $\nu = 0.3$.
See front page for cross-section dimensions.
Calculate the geometrical characteristics.

Egzamin MK(KB) 7.9.2015 zadanie 1

Schemat zastępczy / Primary structure




Symetria / Symmetry

$$\left.\begin{aligned} \lambda^4 &= 3(1-\nu^2)\left(\frac{a}{h_1}\right)^2 \\ C &= \frac{Eh}{1-\nu^2} \\ D &= \frac{Eh^3}{12(1-\nu^2)} \end{aligned}\right\} \text{WALEC CYLINDER}$$

$$\left.\begin{aligned} C &= \frac{3Eh}{1-\nu^2} \\ D &= \frac{27Eh^3}{12(1-\nu^2)} \end{aligned}\right\} \text{PŁYTA PLATE}$$

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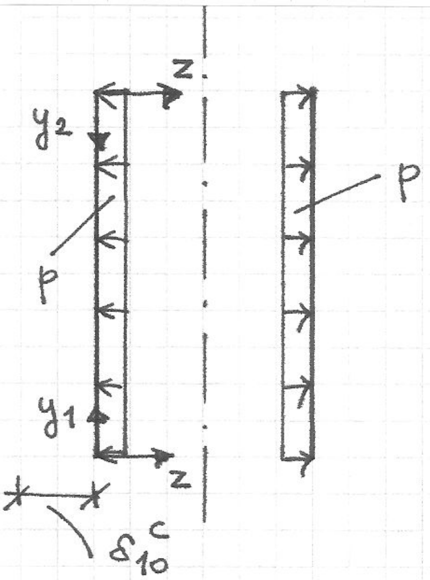
Ⓐ Walec / Cylinder

- Stan bezzgięciowy (stan "0") / Non-bending state ("0"-th state)




$$\xi_1 = \frac{y_1}{a} \qquad \xi_2 = \frac{y_2}{a}$$

$$w(\xi_i) = A_1 e^{-\lambda\xi_i}\cos(\lambda\xi_i) + A_2 e^{-\lambda\xi_i}\sin(\lambda\xi_i) + w_0 \quad i = 1,2$$

$$\lambda^4 = 3(1-\nu^2)\left(\frac{a}{h}\right)^2$$

$$w_0 = -\frac{pa^2}{Eh_1}$$

$$\delta_{10}^c = -w_0 = \frac{pa^2}{Eh_1}$$

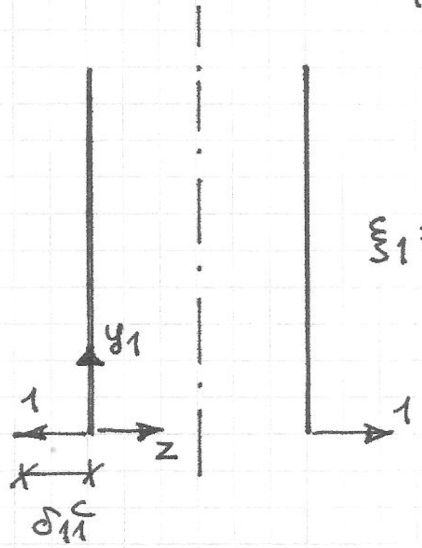
$$M_{10}^c(\xi_i) = 0 \qquad M_{20}^c(\xi_i) = 0$$

$$N_{10}^c(\xi_i) = -\frac{Eh_1}{a}w(\xi_i) = pa \qquad N_{20}^c = \frac{1}{2}pa$$

patrz PŁYTA

see PLATE

- Zaburzenie $X_1 = 1$ / Perturbation $X_1 = 1$




$$\xi_1 = \frac{y_1}{a}$$

Rozpatrujemy zaburzenie na dolnej krawędzi. Zaburzenie na górnej krawędzi wywołuje efekt symetryczny.
Here we consider the perturbation applied to the lower edge. Perturbation applied to the top edge cause symmetric effect.

$$w(\xi_1) = e^{-\lambda \xi_1}\left[A_1 \cos(\lambda \xi_1) + A_2 \sin(\lambda \xi_1)\right]$$

war. brzegowe / boundary conditions

$$Q_2(0) = 1 \quad , \quad M_2(0) = 0 \longmapsto A_1, A_2 \qquad\qquad \xi_2 = \frac{H}{a} - \xi_1$$

$$M_{11}^c(\xi_i) = \nu M_{21}^c(\xi_i) \qquad\qquad \delta_{11}^c = \frac{2a\lambda}{Eh_1}$$

$$M_{21}^c(\xi_i) = \frac{a}{\lambda} \cdot e^{-\lambda \xi_i} \sin(\lambda \xi_i) \qquad\qquad \xi_i \in \{\xi_1, \xi_2\}$$

$$N_{11}^c(\xi_i) = 2\lambda \cdot e^{-\lambda \xi_i} \cos(\lambda \xi_i) \qquad\qquad N_{21}^c(\xi_i) = 0$$

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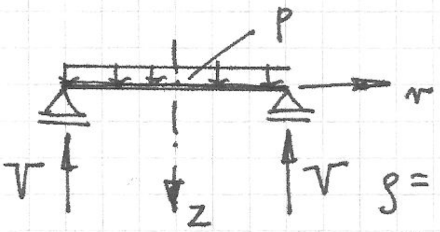
## (B) PŁYTA / PLATE

- Stan "0"
  "0"-th state

Rozpatrujemy płytę dolną. Zachowanie się płyty górnej jest symetryczne.
We consider the lower plate only. The upper plate bends symmetrically.




$$\rho = \frac{r}{a}$$

$$V = N_{20}^c = \frac{p \cdot \pi a^2}{2\pi a} = \frac{1}{2} pa$$

$$w(\rho) = A_1 + A_2 \rho^2 + \frac{qR^4}{64D} \rho^4$$

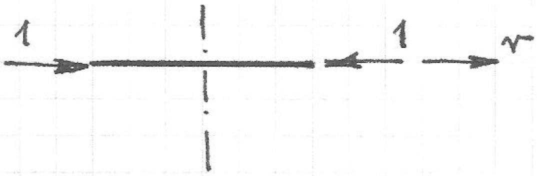
warunki brzegowe / boundary conditions

$$\left.\begin{matrix} w(1) = 0 \\ M_2(1) = 0 \end{matrix}\right| \longmapsto A_1, A_2$$

$$M_{10}^p(\rho) = \frac{pa^2}{16}\left[(3+\nu)(1-\rho^2) + 2(1-\nu)\rho^2\right]$$

$$\delta_{10}^p = 0 \qquad\qquad M_{20}^p(\rho) = \frac{pa^2}{16}(3+\nu)(1-\rho^2)$$

• Zaburzenie $X_1 = 1$ / Perturbation $X_1 = 1$




$$\varrho = \frac{r}{a}$$

$$u(\varrho) = A_1 \varrho$$

war. brzegowe / boundary conditions

$$N_2(1) = -1 \longrightarrow A_1 \qquad \delta_{11}^{p} = -u(1) = \frac{a(1-\nu)}{Eh_2}$$

$$M_{21}^{p}(\varrho) = 0 \qquad M_{22}^{p}(\varrho) = 0$$

$$N_{11}^{p}(\varrho) = -1 \qquad N_{21}^{p}(\varrho) = -1$$

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Ⓒ OBLICZENIE $X_1$ / CALCULATION OF $X_1$

$$\delta_{11} = \delta_{11}^{c} + \delta_{11}^{p} = \frac{2a\lambda}{Eh_1} + \frac{a(1-\nu)}{Eh_2} = \frac{2a\lambda}{Eh} + \frac{a(1-\nu)}{3Eh}$$

$$\delta_{10} = \delta_{10}^{c} = \frac{pa^2}{Eh_1} = \frac{pa^2}{Eh}$$

$$X_1 = -\frac{\delta_{10}}{\delta_{11}}$$

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Ⓓ OBLICZENIE $N_1, N_2, M_1, M_2$

CALCULATION OF $N_1, N_2, M_1, M_2$

Walec / cylinder

$$N_1^{c}(\xi_1) = \left[N_{11}^{c}(\xi_1) + N_{11}^{c}\left(\frac{H}{a} - \xi_1\right)\right] \cdot X_1 + \underbrace{N_{10}^{c}(\xi_1)}_{=0}$$

$$N_2^{c}(\xi_1) = N_{20}^{c}(\xi_1)$$

$$M_1^{c}(\xi_1) = \nu M_2^{c}(\xi_1)$$

$$M_2^{c}(\xi_1) = \left[M_{21}^{c}(\xi_1) + M_{21}^{c}\left(\frac{H}{a} - \xi_1\right)\right] \cdot X_1 + \underbrace{M_{20}^{c}(\xi_1)}_{=0}$$

Płyta / plate

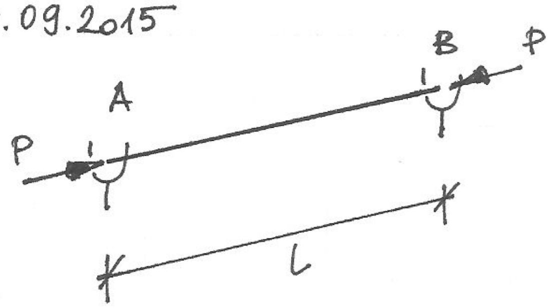
$$N_1^{p}(\varrho) = N_{11}^{p}(\varrho) \cdot X_1$$

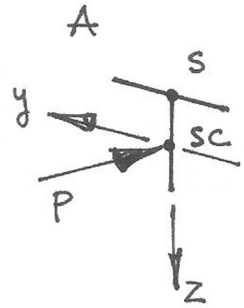
$$N_2^{p}(\varrho) = N_{21}^{p}(\varrho) \cdot X_1$$

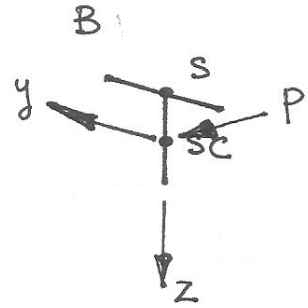
$$M_1^{p}(\varrho) = M_{10}^{p}(\varrho)$$

$$M_2^{p}(\varrho) = M_{20}^{p}(\varrho)$$

Problem #2
Zadanie 2
7.09.2015

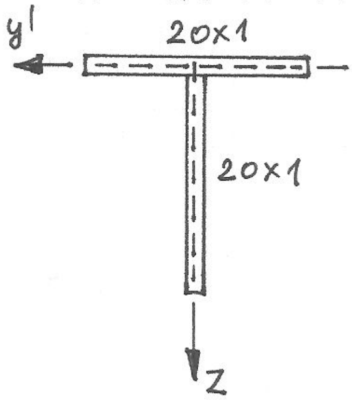



Cechą charakterystyczną przekroju teowego jest $\omega = 0$ oraz $J_\omega = 0$.
The main feature of a T-section is $\omega = 0$ and $J_\omega = 0$.

Należy wyznaczyć pozostałe charakterystyki geometryczne.
We need to calculate all other geometrical characteristics.



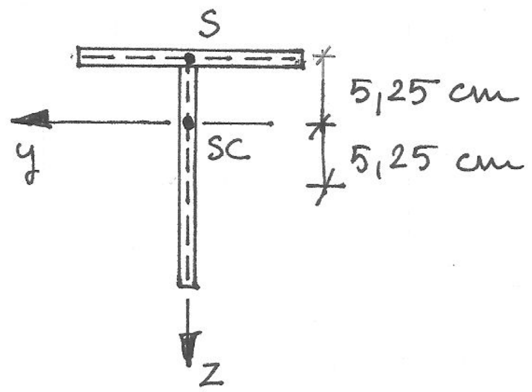

$$A = 40 \text{ cm}^2$$

$$J_S = \frac{1}{3} \cdot 20 \cdot 1^3 \cdot 2 = 13{,}33 \text{ cm}^4$$

$$S_z = 0$$

$$S_{y'} = 1 \cdot 20 \cdot 10{,}5 = 210 \text{ cm}^3$$

$$z_{SC} = \frac{S_{y'}}{A} = 5{,}25 \text{ cm}$$




$$J_z = \frac{1}{12}\left[1 \cdot 20^3 + 20 \cdot 1^3\right] = 668{,}33 \text{ cm}^4$$

$$J_y = \left[\frac{1}{12} \cdot 20 \cdot 1^3 + 20 \cdot (5{,}25)^2 + \frac{1}{12} \cdot 1 \cdot 20^3 + 20 \cdot (5{,}25)^2\right] = 1770{,}83 \text{ cm}^4$$

$$z_S = -5{,}25 \text{ cm} \qquad y_S = 0$$

$$E = 205 \text{ GPa} = 20500 \ \frac{\text{kN}}{\text{cm}^2} \qquad E_1 = \frac{E}{1-\nu^2} = 22527 \ \frac{\text{kN}}{\text{cm}^2}$$

$$\nu = 0{,}3 \qquad G = \frac{E}{2(1+\nu)} = 7885 \ \frac{\text{kN}}{\text{cm}^2}$$

$$l = 300 \text{ cm}$$

$$(r_0)^2 = \frac{J_z + J_y}{A} + (z_S)^2 + (y_S)^2 = 76{,}34 \text{ cm}^2$$

$$\beta_z = \left(\frac{z_s}{r_o}\right)^2 = 0{,}36$$

$$P_z = \frac{\pi^2 E_1 J_z}{l^2} = 1651 \text{ kN}$$

$$P_y = \frac{\pi^2 E_1 J_y}{l^2} = 4375 \text{ kN}$$

$$P_s = \frac{1}{(r_o)^2}\left[ G J_s + \frac{\pi^2 E_1 J_\omega}{l^2} \right] = 1377 \text{ kN}$$

Teraz obliczamy $P_{kr}$ (krytyczne).

Now we calculate $P_{kr}$ (critical value).

$$\det[\mathbb{A}(P)] = 0 \longrightarrow P = P_{kr}$$

$$\mathbb{A}(P) = \begin{bmatrix} P-P_z & 0 & z_s P \\ 0 & P-P_y & 0 \\ z_s P & 0 & (r_o)^2(P-P_s) \end{bmatrix}$$

$$\det[\mathbb{A}(P)] = (r_o)^2 \cdot (P-P_y) \cdot W_2(P) = (r_o)^2 \cdot (P-P_y) \cdot (P-P_1) \cdot (P-P_2) \cdot (1-\beta_z)$$

$$W_2(P) = (1-\beta_z)P^2 - (P_z+P_s)P + P_z P_s$$

$P_1, P_2$ – pierwiastki wielomianu $W_2(P)$ / roots of the polynomial $W_2(P)$

$$P_1 = \frac{P_z + P_s - \sqrt{\Delta}}{2(1-\beta_z)}, \quad P_2 = \frac{P_z + P_s + \sqrt{\Delta}}{2(1-\beta_z)}, \quad \Delta = (P_z+P_s)^2 - 4(1-\beta_z)P_z P_s$$

$P_1 = 935$ kN

$P_2 = 3803$ kN

$$P_{kr} = \min\{P_1, P_2, P_y\} = P_1 = 935 \text{ kN}.$$