EGZAMIN MK (MK3) KB studia mgr stacj. dnia 7 IX 2016

NAZWISKO Imię		
Nr albumu		Ocena z ćwiczeń projektowych
ocena zadania 1	ocena zadania 2	Ocena z egzaminu po ustnym
		Ocena łączna, data, podpis

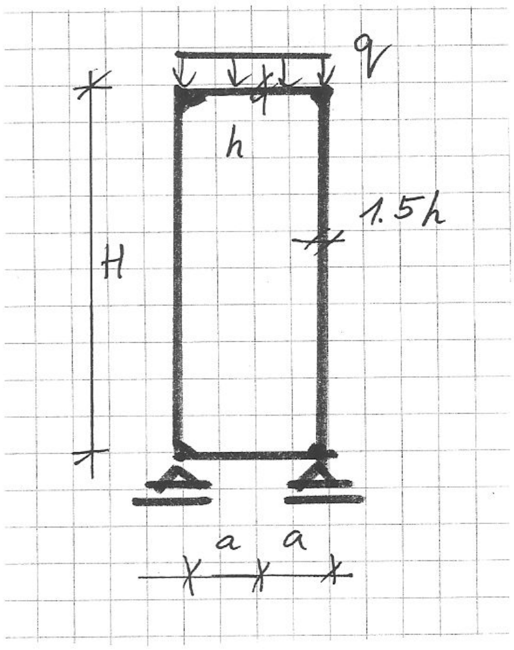
Zadanie 1.

Dany jest wysoki zbiornik walcowy, obciążony jak na rysunku.
Znaleźc wykresy momentów zginających M_1, M_2.

Dane: E, $\nu = 0.2$

$h = \frac{1}{10} a$

$H = 5a$



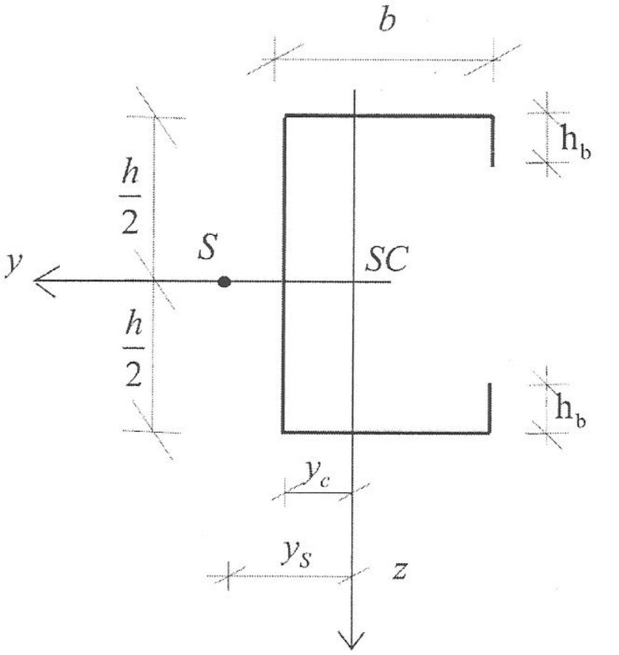
Zadanie 2.

Znaleźć siłę krytyczną P
(przyłożoną w środku ciężkości SC przekroju)
wyboczenia giętno-skrętnego pręta cienkościennego
o danym przekroju, podpartego widełkowo,
o długości $l = 3.0$ m. Dane dotyczące przekroju:

h = 40 cm
b = 20 cm
hb =4 cm
grubość g=1cm

yc = 6,36 cm
ys = 15,68 cm

$A=88cm^2$
$Jy=23936cm^4$
$Jz=4970cm^4$
$J\omega=1606085cm^6$
$Js=29,3cm^4$



Pozostałe charakterystyki wyznaczyć samodzielnie.

Exam on the Mechanics of Structures
11.02.2016

PROBLEM #1
Consider a cylindrical shell with top and bottom circular plates and loaded by a uniform loading q applied to a top plate.
Assume that the shell is long.

Calculate the bending moments M_1, M_2 in a cylinder and plates.

See front page for the Poisson ratio ν and shell dimensions.

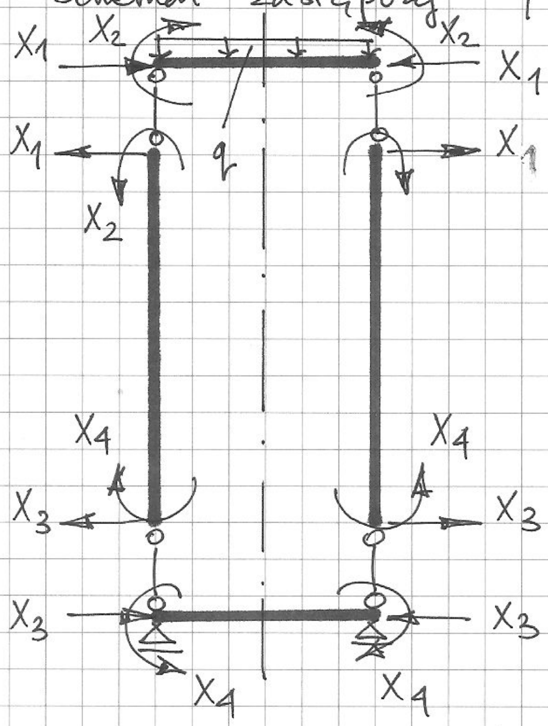
PROBLEM #2
Consider a thin-walled beam of length $l = 3.0\,m$, fork-supported and loaded by a force P applied at cross-section's centroid SC.

Calculate the value of the force P critical for flexural-torsional buckling.

See front page for cross-section dimensions and geometrical characteristics $A, J_y, J_z, J_\omega, J_s$.
Assume the thickness of a section $g = 1cm$.
Calculate the remaining characteristics, if necessary.

Zadanie 1 Problem #1

Schemat zastępczy / Primary structure



Walec i płyta dolna są nieobciążone, w związku z czym przemieszczenia $\delta_{30} = \delta_{40} = 0$. Co za tym idzie $X_3 = X_4 = 0$.

Note that the cylinder and the bottom plate are not loaded by external load. Hence the displacements $\delta_{30} = \delta_{40} = 0$ and consequently $X_3 = X_4 = 0$.

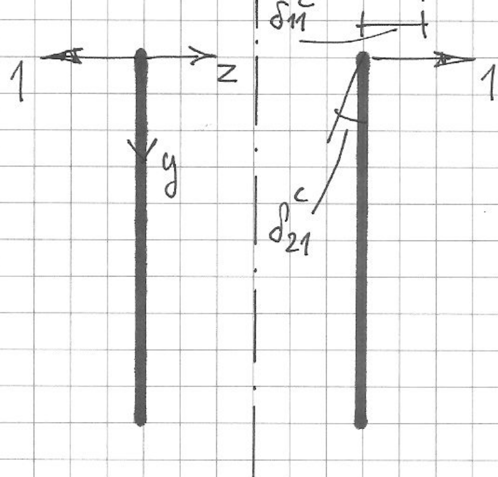
Stałe materiałowe / Material constants

walec / cylinder $\quad \lambda^4 = 3(1-\nu^2)\left(\frac{a}{1{,}5h}\right)^2, \; C_1 = \frac{E(1{,}5h)}{1-\nu^2}, \; D_1 = \frac{(1{,}5h)^2}{12} C_1$

płyta / plate $\quad C_2 = \frac{Eh}{1-\nu^2}, \; D_2 = \frac{h^2}{12} C_2$

Ⓐ WALEC / CYLINDER

- Zaburzenie $X_1 = 1$ / Perturbation $X_1 = 1$



$\xi = \frac{y}{a}$

$$w(\xi) = e^{-\lambda\xi}\left[A_1 \cos(\lambda\xi) + A_2 \sin(\lambda\xi)\right]$$

warunki brzegowe / boundary conditions

$$\left.\begin{array}{l} M_2(0) = 0 \\ Q_2(0) = 1 \end{array}\right| \longrightarrow A_1, A_2$$

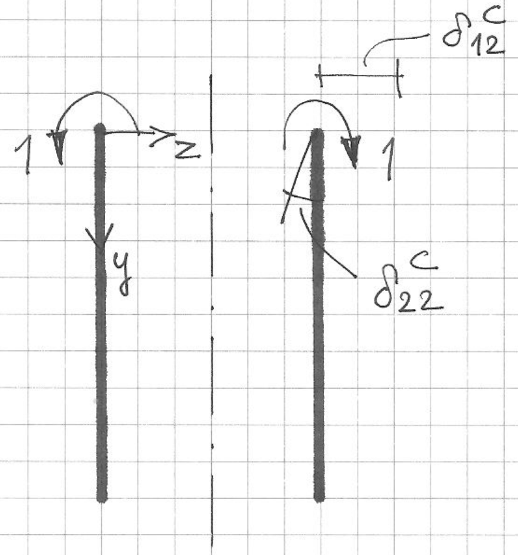
$$\delta_{11}^c = -w(0) = \frac{2a\lambda}{E(1{,}5h)}$$

$$\delta_{21}^c = \chi_2(0) = \frac{2\lambda^2}{E(1{,}5h)}$$

$$M_{11}^c(\xi) = \nu M_{21}^c(\xi)$$

$$M_{21}^c(\xi) = \frac{a}{\lambda} e^{-\lambda\xi} \sin(\lambda\xi)$$

• Zaburzenie $X_2=1$ / Perturbation $X_2=1$



$$\xi=\frac{y}{a}$$

$w(\xi)$ – jak w przypadku obciążenia $X_1=1$
like in the $X_1=1$ loading case

warunki brzegowe / boundary conditions

$$\left.\begin{array}{l} M_2(0)=1 \\ Q_2(0)=0 \end{array}\right| \longrightarrow A_1, A_2$$

$$\delta_{12}^{c}=-w(0)=\frac{2\lambda^2}{E\cdot(1{,}5h)}$$

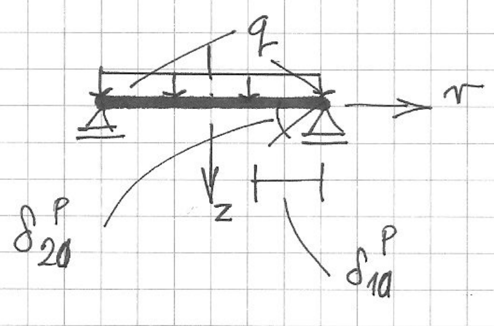
$$\delta_{22}^{c}=\chi_2(0)=\frac{4\lambda^3}{Ea\cdot(1{,}5h)}$$

$$M_{12}^{c}(\xi)=\nu M_{22}^{c}(\xi)$$

$$M_{22}^{c}(\xi)=e^{-\lambda\xi}\left[\cos(\lambda\xi)-\sin(\lambda\xi)\right]$$

(B) PŁYTA GÓRNA / TOP PLATE

• Stan "0" / The "0"-th state



$$\rho=\frac{r}{a}$$

$$w(\rho)=A_1+A_2\rho^2+\frac{qa^4}{64D_2}\rho^4$$

war. brzegowe / boundary conditions

$$\left.\begin{array}{l} w(1)=0 \\ M_2(1)=0 \end{array}\right| \longrightarrow A_1, A_2$$

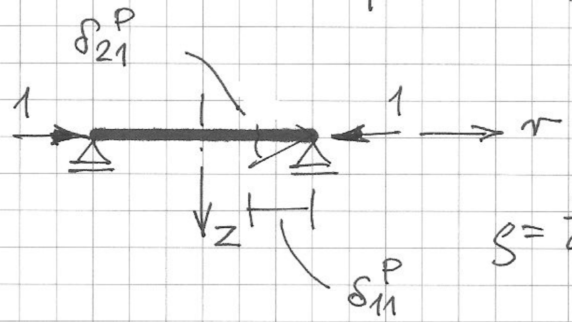
$$\delta_{10}^{P}=0$$

$$\delta_{20}^{P}=-\chi_2(1)=\frac{qa^3}{8D_2(1+\nu)}$$

$$M_{10}^{P}(\rho)=\frac{qa^2}{16}\left[(3+\nu)(1-\rho^2)+2(1-\nu)\rho^2\right]$$

$$M_{20}^{P}(\rho)=\frac{qa^2}{16}\left[(3+\nu)(1-\rho^2)\right]$$

- Zaburzenie $X_1 = 1$ / Perturbation $X_1 = 1$



$\varrho = \frac{r}{a}$

$u(\varrho) = A_1 \varrho$

war. brzegowe / boundary conditions

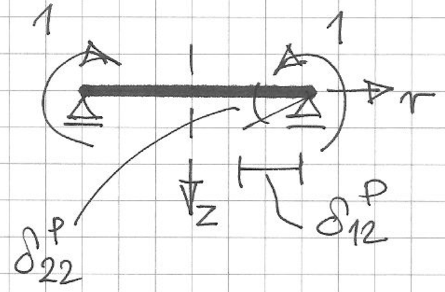
$N_2(1) = -1 \longrightarrow A_1$

$\delta_{11}^P = -u(1) = \frac{a}{C_2(1+\nu)}$

$M_{11}^P = M_{21}^P = 0$

$\delta_{21}^P = 0$

- Zaburzenie $X_2 = 1$ / Perturbation $X_2 = 1$



$\varrho = \frac{r}{a}$ $\quad w(\varrho) = A_1 + A_2 \varrho^2$

war. brzegowe / boundary conditions

$\left.\begin{array}{l} w(1) = 0 \\ M_2(1) = 1 \end{array}\right| \longrightarrow A_1, A_2$

$M_{12}^P(\varrho) = 1$

$\delta_{12}^P = 0$

$M_{22}^P(\varrho) = 1$

$\delta_{22}^P = \chi_2(1) = \frac{a}{D_2(1+\nu)}$

Ⓒ Obliczenie X_1, X_2 / Calculation of X_1, X_2

$$\left.\begin{array}{l} \delta_{11} X_1 + \delta_{12} X_2 + \delta_{10} = 0 \\ \delta_{21} X_1 + \delta_{22} X_2 + \delta_{20} = 0 \end{array}\right\} \longrightarrow X_1, X_2$$

$$\delta_{11} = \delta_{11}^C + \delta_{11}^P = \frac{2a\lambda}{1{,}5Eh} + \frac{a}{C_2(1+\nu)}$$

$$\delta_{12} = \delta_{21} = \delta_{12}^P + \delta_{12}^C = \frac{2\lambda^2}{1{,}5Eh}$$

$$\delta_{22} = \delta_{22}^C + \delta_{22}^P = \frac{4\lambda^3}{1{,}5Eah} + \frac{a}{D_2(1+\nu)}$$

Ⓓ CALCULATION OF M_1, M_2

walec / cylinder:

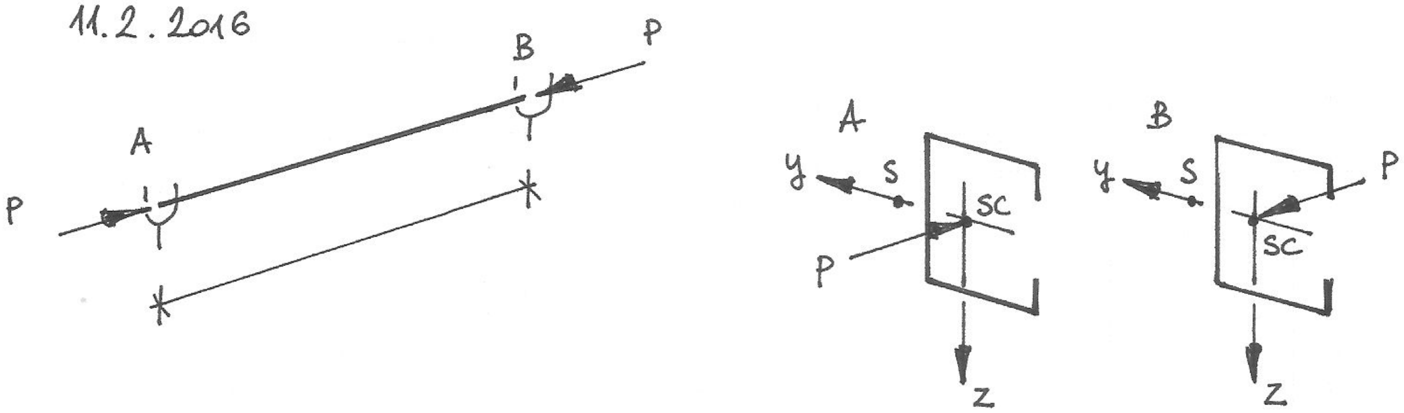
$M_1^C(\xi) = \nu M_2^C(\xi)$, $M_2^C(\xi) = M_{21}^C(\xi) \cdot X_1 + M_{22}^C(\xi) \cdot X_2$

płyta / plate

$M_1^P(\varrho) = M_{12}^P(\varrho) \cdot X_2 + M_{10}^P(\varrho)$

$M_2^P(\varrho) = M_{22}^P(\varrho) \cdot X_2 + M_{20}^P(\varrho)$

Zadanie 2
Problem #2
11.2.2016



Zakładamy, że belka jest wykonana ze stali.
Assume that the beam is made of steel.

$$E = 205\ \text{GPa} = 20500\ \frac{kN}{cm^2} \qquad \nu = 0.3$$

$$E_1 = \frac{E}{1-\nu^2} = 22527\ \frac{kN}{cm^2} \qquad G = \frac{E}{2(1+\nu)} = 7885\ \frac{kN}{cm^2}$$

$$z_s = 0, \quad y_s = 15{,}68\ cm \qquad \beta_z = \left(\frac{z_s}{r_o}\right)^2 = 0$$

$$(r_o)^2 = \frac{J_y + J_z}{A} + (y_s)^2 + (z_s)^2 = 574{,}34\ cm^2 \qquad \beta_y = \left(\frac{y_s}{r_o}\right)^2 = 0{,}43$$

$$P_z = \frac{\pi^2 E_1 J_z}{L^2} = 12278\ kN$$

$$P_y = \frac{\pi^2 E_1 J_y}{L^2} = 59131\ kN$$

$$P_s = \frac{1}{(r_o)^2}\left[G J_s + \frac{\pi^2 E_1 J_\omega}{L^2}\right] = 7310\ kN$$

W przypadku $y_s \neq 0$, $z_s = 0$ macierz $A(P)$ ma postać:
In case of $y_s \neq 0$, $z_s = 0$ the matrix $A(P)$ takes the form:

$$A(P) = \begin{bmatrix} P-P_z & 0 & 0 \\ 0 & P-P_y & -y_s P \\ 0 & -y_s P & (r_o)^2 (P-P_s) \end{bmatrix}$$

Krytyczna wartość $P = P_{kr}$ spełnia warunek:
the critical value $P = P_{kr}$ fulfills:

$$\det[A(P)] = 0$$

$$\det\left[\mathbb{A}(P)\right] = (r_0)^2 \cdot (P - P_z) \cdot W_2(P)$$

$$W_2 = (1-\beta_y)P^2 - (P_S + P_y)P + P_S P_y = (1-\beta_y)(P-P_1)(P-P_2)$$

P_1, P_2 - pierwiastki wielomianu $W_2(P)$ / roots of the polynomial $W_2(P)$

$$P_1 = \frac{P_S + P_y - \sqrt{\Delta}}{2(1-\beta_y)}$$

$$P_2 = \frac{P_S + P_y + \sqrt{\Delta}}{2(1-\beta_y)}$$

$$\Delta = (P_S + P_y)^2 - 4(1-\beta_y)P_S P_y$$

$P_1 = 6918$ kN

$P_2 = 109256$ kN

$$P_{kr} = \min\{P_1, P_2, P_z\} = P_1$$