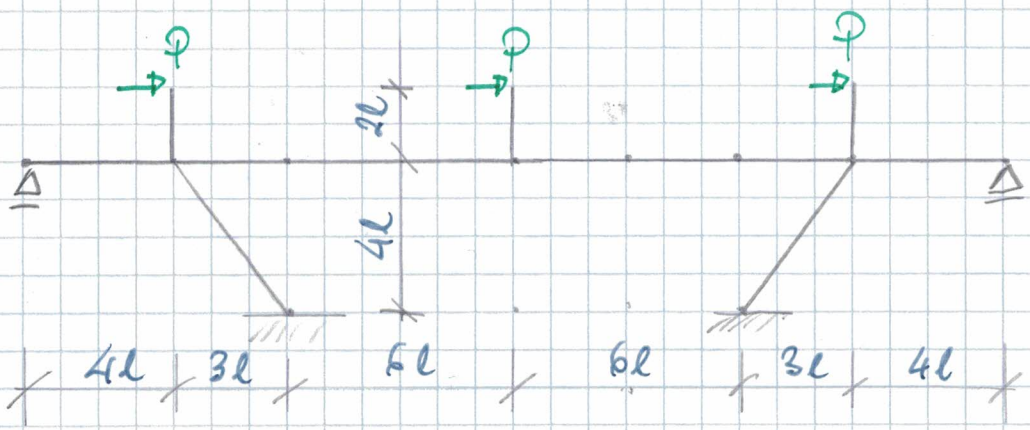
Kolokwium nr 2.2. MK1 r.a. 2024/25

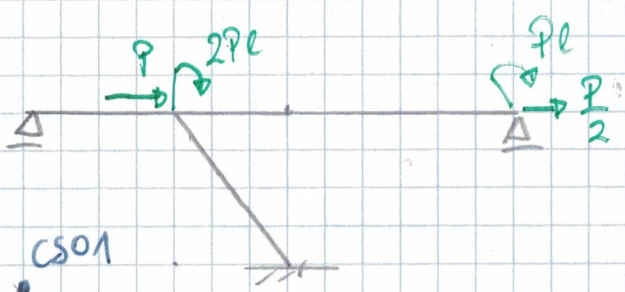


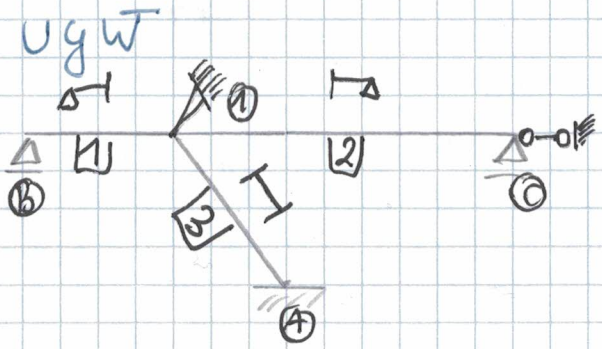
$EJ = const.$

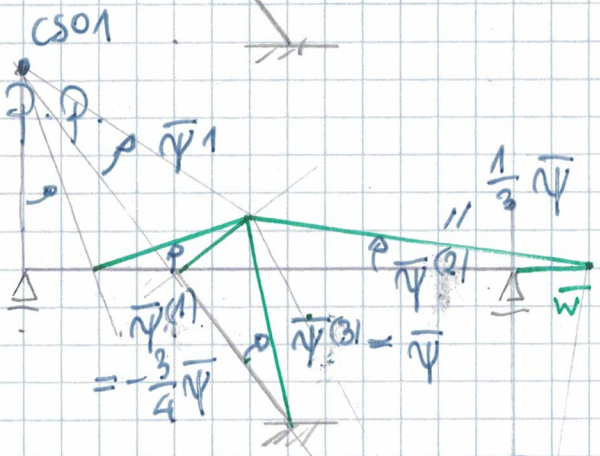
$EA \to \infty$

(M) = ?

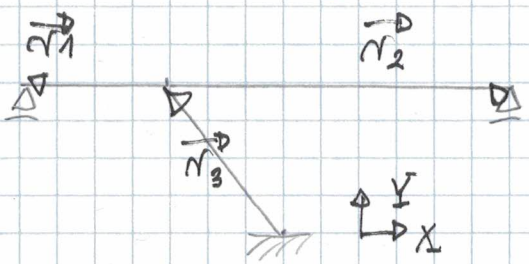
Schemat połówkowy i redukcja cz. stat. wyznaczalnej

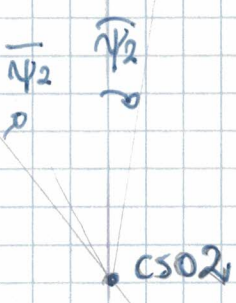






metoda analityczna





$\vec{r}_3 = [-3l, 4l]$

$\vec{r}_1 = [-4l, 0]$

$\vec{r}_2 = [9l, 0]$

$u_y^{(B)}: \quad 0 = 0 + 3l\,\bar{\psi}^{(3)} + 4l\,\bar{\psi}^{(1)}$

$u_y^{(C)}: \quad 0 = 0 + 3l\,\bar{\psi}^{(3)} - 9l\,\bar{\psi}^{(2)}$

$\bar{\psi}^{(3)} = \bar{\psi} \qquad \bar{\psi}^{(1)} = -\frac{3}{4}\bar{\psi} \qquad \bar{\psi}^{(2)} = \frac{1}{3}\bar{\psi}$

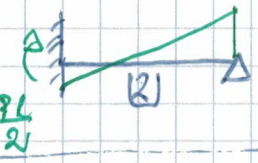
r. n. MP

$$\Phi_1^{(1)} + \Phi_1^{(2)} + \Phi_1^{(3)} = 2Pl \quad (1)$$

$$\Phi_1^{(1)}\left(-\frac{3}{4}\bar{\psi}\right) + \left(\Phi_1^{(3)} + \Phi_A^{(3)}\right)\bar{\psi} + \Phi_1^{(2)}\left(\frac{1}{3}\bar{\psi}\right) + \bar{L}_{obc} = 0$$

$$\bar{L}_{obc} = Pl\cdot\frac{1}{3}\bar{\psi} + \frac{P}{2}\cdot\bar{\psi}\cdot 4l + P\cdot\bar{\psi}\cdot 4l = 6\frac{1}{3}Pl\bar{\psi}$$

$$\frac{3}{4}\Phi_1^{(1)} - \left(\Phi_1^{(3)} + \Phi_A^{(3)}\right) - \frac{1}{3}\Phi_1^{(2)} = 6\frac{1}{3}Pl \quad (2)$$

Momenty wyjściowe 

$\Phi_1^{(02)} = \frac{Pl}{2}$

WT

1) 

$$\Phi_1^{(1)} = \frac{3EJ}{4l}\left[\varphi_1 + \frac{3}{4}\psi\right] = \frac{EJ}{l}\left[\frac{3}{4}\varphi_1 + \frac{9}{16}\psi\right] \quad \left(\frac{3}{4}\right)$$

---

2) 

$$\Phi_1^{(2)} = \frac{3EJ}{9l}\left[\varphi_1 - \frac{1}{3}\psi\right] + \frac{Pl}{2} = \frac{EJ}{l}\left[\frac{1}{3}\varphi_1 - \frac{1}{9}\psi\right] + \frac{Pl}{2} \quad \left(-\frac{1}{3}\right)$$

---

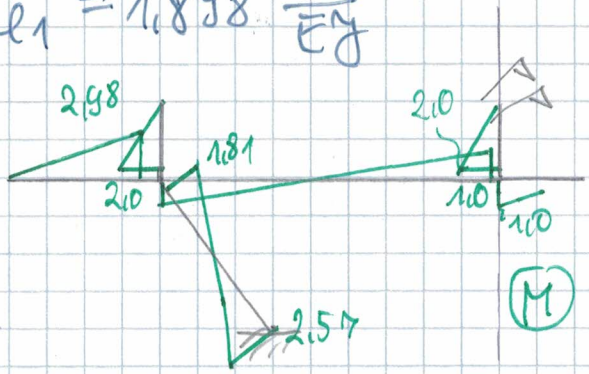
3) 

$$\Phi_1^{(1)} = \frac{2EJ}{5l}\left[2\varphi_1 - 3\psi\right] = \frac{EJ}{l}\left[\frac{4}{5}\varphi_1 - \frac{6}{5}\psi\right] \quad (-1)$$

$$\Phi_A^{(1)} = \frac{EJ}{l}\left[\frac{2}{5}\varphi_1 - \frac{6}{5}\psi\right]$$

$$\frac{EJ}{l}\begin{bmatrix} \frac{3}{4}+\frac{1}{3}+\frac{4}{5} & \frac{9}{16}-\frac{1}{9}-\frac{6}{5} \\ \frac{9}{16}-\frac{1}{9}-\frac{6}{5} & \frac{27}{64}+\frac{1}{27}+\frac{12}{5} \end{bmatrix}\begin{bmatrix} \varphi_1 \\ \psi \end{bmatrix} = \begin{bmatrix} 2-\frac{1}{2} \\ 6\frac{1}{3}+\frac{1}{6} \end{bmatrix} Pl$$

$$\varphi_1 = 1{,}898\,\frac{Pl^2}{EJ} \qquad \psi = 2{,}771\,\frac{Pl^2}{EJ}$$

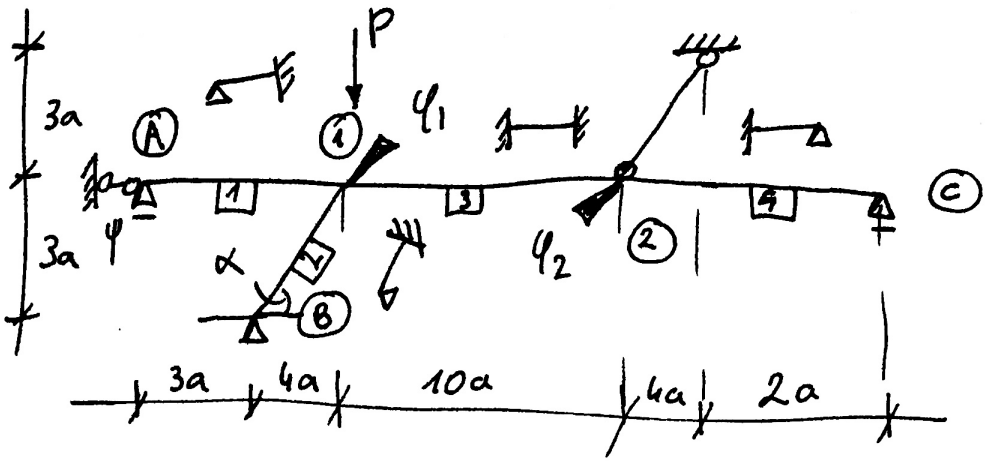


(M) [Pl]

K2.2 15 I 2025

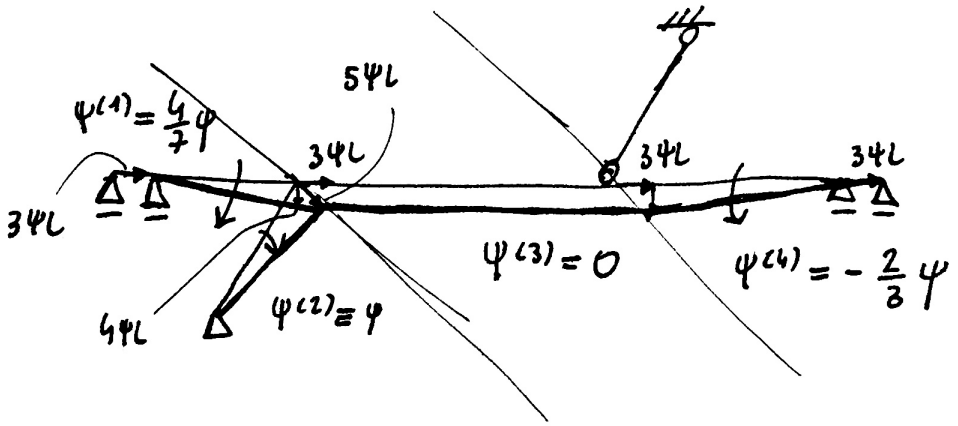
$EA = \infty$, $EJ = const.$; $M = ?$

SCHEMAT POŁÓWKOWY + UGW



$$q = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \psi \end{bmatrix}$$

PLAN PRZEM. (CIĘCIWY PRĘTÓW)



$$\begin{cases} \psi^{(1)} = \frac{4}{7}\psi \\ \psi^{(2)} \equiv \psi \\ \psi^{(3)} = 0 \\ \psi^{(4)} = -\frac{2}{3}\psi \end{cases}$$

RRMP:

$$\bar{L}_{\substack{OBC \\ ZEW}} = 4\psi L \cdot P$$

1) $\phi_1^{(1)} + \phi_1^{(2)} + \phi_1^{(3)} = 0$

2) $\phi_2^{(3)} + \phi_2^{(4)} = 0$

3) $\phi_1^{(1)} \cdot \frac{4}{7}\bar{\psi} + \phi_1^{(2)} \cdot \bar{\psi} + (\phi_1^{(3)} + \phi_2^{(3)}) \cdot 0 + \phi_2^{(4)} \cdot (-\frac{2}{3}\bar{\psi}) +$

$+ \bar{L}_{\substack{OBC \\ ZEW}} = 0$

WZORY TRANSFORMACYJNE:

$$\phi_1^{(1)} = \frac{3EJ}{7a}\left[\varphi_1 - \psi^{(1)}\right] \qquad \phi_1^{(3)} = \frac{2EJ}{10a}\left[2\varphi_1 + \varphi_2 - 3\psi^{(3)}\right]$$

$$\phi_1^{(2)} = \frac{3EJ}{5a}\left[\varphi_1 - \psi^{(2)}\right] \qquad \phi_2^{(3)} = \frac{2EJ}{10a}\left[\varphi_1 + 2\varphi_2 - 3\psi^{(3)}\right]$$

$$\phi_2^{(4)} = \frac{3EJ}{6a}\left[\varphi_2 - \psi^{(4)}\right]$$

· po podstawieniu wzorów transf. do RRMP i przyjęciu $\bar{\psi} = -1$ otrzymamy:

$$\frac{EJ}{a}\begin{bmatrix} 10/7 & 1/5 & -207/245 \\ 1/5 & 9/10 & 1/3 \\ -207/245 & 1/3 & 0{,}962 \end{bmatrix}\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \psi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} Pa$$

$$\rightarrow \varphi_1 = 11{,}627\ \frac{Pa^2}{EJ} \qquad \varphi_2 = -9{,}070\ \frac{Pa^2}{EJ} \qquad \psi = 17{,}513\ \frac{Pa^3}{EJ}$$