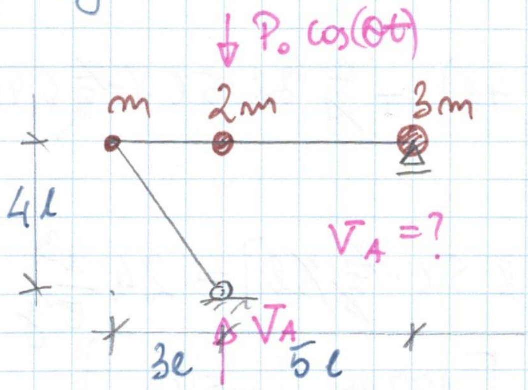
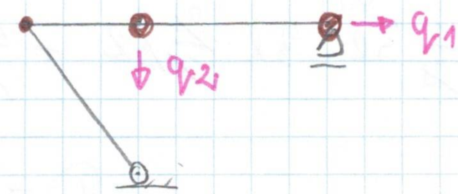
Kolokwium 2.1. MK 2 7 V 2026

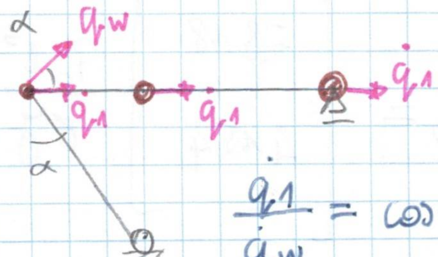
$EJ = \text{const.}$ $EA \to \infty$ $\theta = \frac{1}{10}\sqrt{\frac{EJ}{m l^3}}$

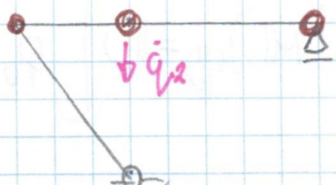


2 st. sw. dyn.



Plany prędkości





$$\frac{\dot{q}_1}{\dot{q}_w} = \cos\alpha = \frac{4}{5}$$

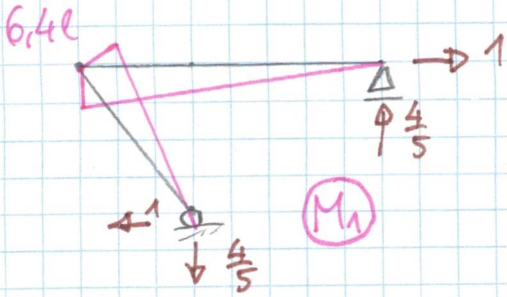
$$\dot{q}_w = \frac{5}{4}\dot{q}_1$$

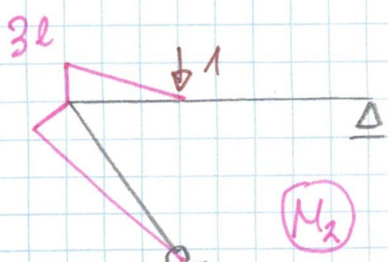
$$E_k = \frac{1}{2}\left[m\left(\frac{5}{4}\dot{q}_1\right)^2 + 2m\left(\dot{q}_1^2 + \dot{q}_2^2\right) + 3m\,\dot{q}_1^2 \right]$$

$=$

Macierz mas $M = m\begin{bmatrix} 6{,}562 & 0 \\ 0 & 2 \end{bmatrix}$

Macierz podatności





$$d_{11} = \frac{1}{EJ}\left[\frac{1}{\cancel{2}} 6{,}4l \cdot 8l \cdot \frac{\cancel{2}}{3} 6{,}4l + \frac{1}{\cancel{2}} 6{,}4l \cdot 5l \frac{\cancel{2}}{3} 6{,}4l\right] =$$

$$= 177{,}49 \frac{l^3}{EJ}$$

$$d_{12} = \frac{1}{EJ}\left[\frac{1}{2} 3l \cdot 3l \cdot \left(-\frac{2}{3} 6{,}4l - \frac{1}{3} 4l\right) + \frac{1}{\cancel{2}} \cancel{3}l \cdot 5l \cdot \left(-\frac{\cancel{2}}{\cancel{3}} 6{,}4l\right)\right] =$$

$$= -57{,}2 \frac{l^3}{EJ}$$

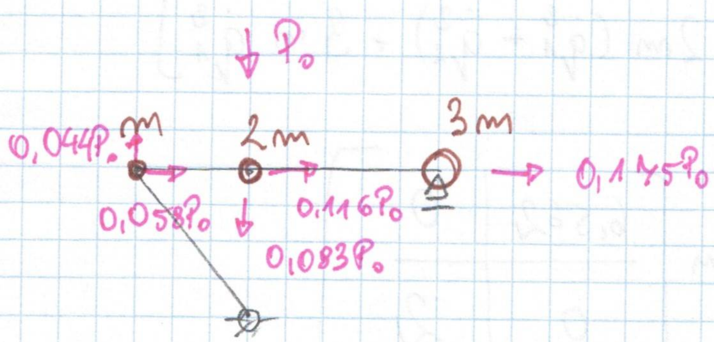
$$d_{22} = \frac{1}{EJ}\left[\frac{1}{\cancel{2}} 3l \cdot 3l \cdot \frac{\cancel{2}}{\cancel{3}} \cancel{3}l + \frac{1}{\cancel{2}} 3l \cdot 5l \cdot \frac{\cancel{2}}{\cancel{3}} \cancel{3}l\right] = 24 \frac{l^3}{EJ}$$

$$\mathbb{D} = \frac{l^3}{EJ} \begin{bmatrix} 177{,}49 & -57{,}2 \\ -57{,}2 & 24 \end{bmatrix}$$

Równanie ruchu

$$(\mathbb{1} - \Theta^2 \mathbb{D} \mathbb{M}) q_0 = \mathbb{D} \begin{bmatrix} 0 \\ P_0 \end{bmatrix} \qquad q_0 = \begin{bmatrix} 5{,}818 \\ 4{,}154 \end{bmatrix} \frac{P_0 l^3}{EJ}$$

Siły bezwładności działające na ramę

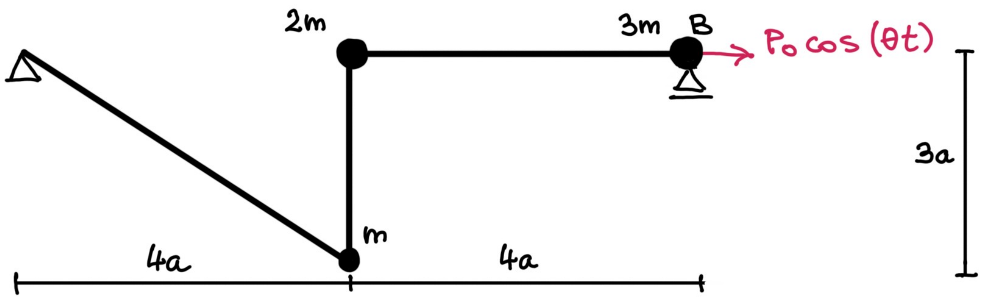


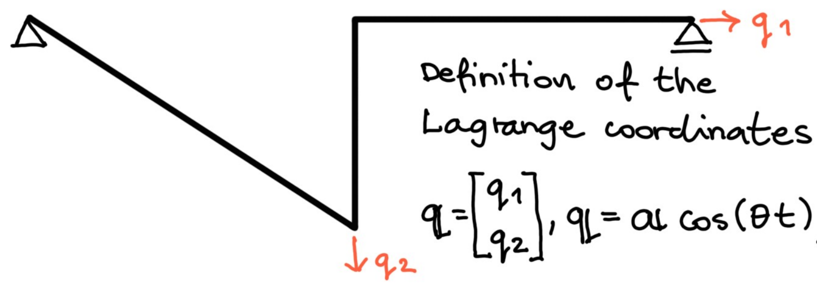
$V_A = 0{,}73 P$

Test 2.1 2025/26 (MoS 2 course)

Compute the amplitude of V_B.

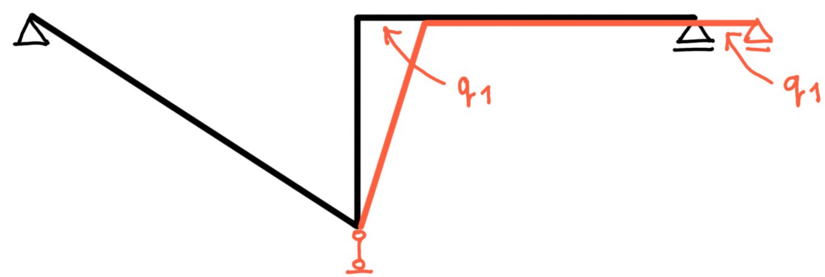
$\theta = \frac{1}{10}\sqrt{\frac{EJ}{ma^3}}$, $\mu = 0$, $EA = \infty$, EJ – const.

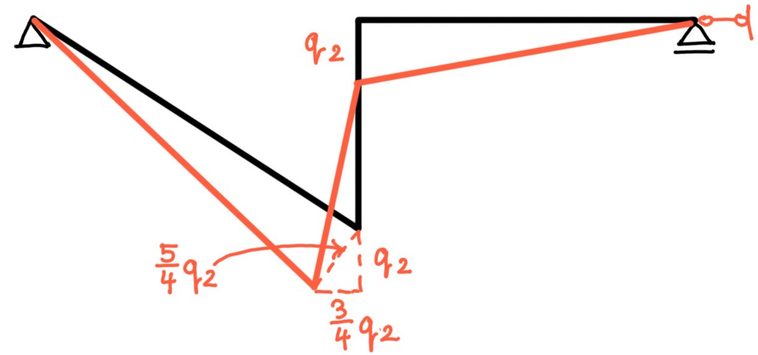


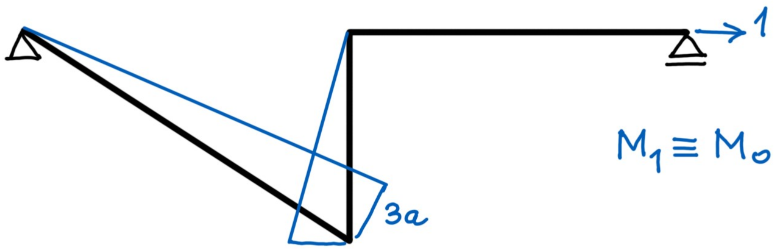


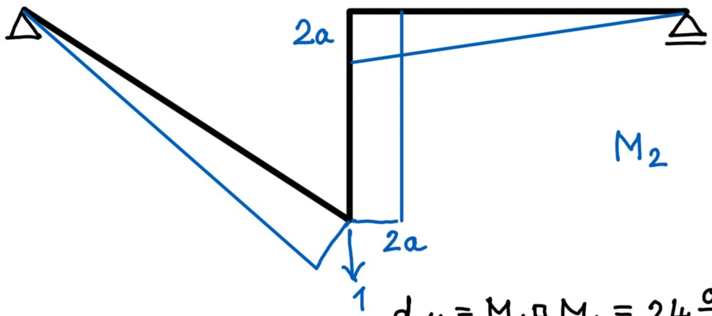
Definition of the Lagrange coordinates

$\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$, $\mathbf{q} = \mathbf{a} \cos(\theta t)$, $\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$









$$E_k = \frac{1}{2}\left\{ m\left(\frac{5}{4}\dot{q}_2\right)^2 + 2m\left[(\dot{q}_1)^2 + (\dot{q}_2)^2\right] + 3m(\dot{q}_1)^2 \right\} =$$

$$= \frac{1}{2}\dot{\mathbf{q}}^T \mathbb{M} \dot{\mathbf{q}} \Rightarrow \mathbb{M} = \begin{bmatrix} 5m & 0 \\ 0 & \frac{57}{16}m \end{bmatrix}$$

$$d_{11} = M_1 \square M_1 = 24\frac{a^3}{EJ}$$

$$d_{12} = d_{21} = M_1 \square M_2 = -19\frac{a^3}{EJ}$$

$$d_{22} = M_2 \square M_2 = 24\frac{a^3}{EJ}$$

$$d_{10} = M_1 \square M_0 = 24\frac{a^3}{EJ}$$

$$d_{20} = M_2 \square M_0 = -19\frac{a^3}{EJ}$$

$$\mathbb{D} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \qquad \mathbf{d}_0 = \begin{bmatrix} d_{10} \\ d_{20} \end{bmatrix}$$

From the equation of motion:

$$\left(\mathbb{I} - \theta^2 \mathbb{D}\mathbb{M}\right)\mathbf{a} = \mathbf{d}_0 P_0 ,$$

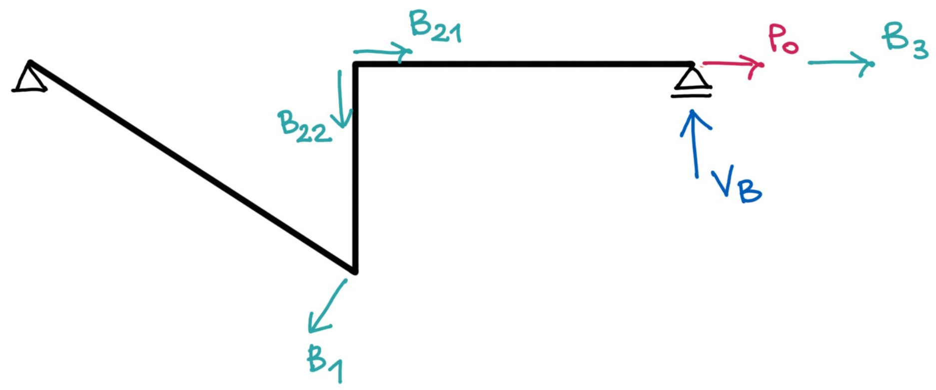
where

$$\mathbb{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} .$$

Then,

$$\mathbf{a} = \left(\mathbb{I} - \theta^2 \mathbb{D}\mathbb{M}\right)^{-1} \mathbf{d}_0 P_0$$

$$= \begin{bmatrix} -24{,}315 \\ 28{,}272 \end{bmatrix} \frac{P_0 a^3}{EJ}$$



$$B_1 = m\theta^2 \cdot \tfrac{5}{4} a_2 = 0{,}353\, P_0$$

$$B_{21} = 2m\theta^2 a_1 = -0{,}486\, P_0$$

$$B_{22} = 2m\theta^2 a_2 = 0{,}565\, P_0$$

$$B_3 = 3m\theta^2 a_1 = -0{,}729\, P_0$$

$$V_B \cdot 8a - B_{22} \cdot 4a - B_1 \cdot 5a = 0$$

$$V_B = \frac{1}{2} B_{22} + \frac{5}{8} B_1$$

$$= 0{,}504\, P_0$$

G. Dzierżanowski