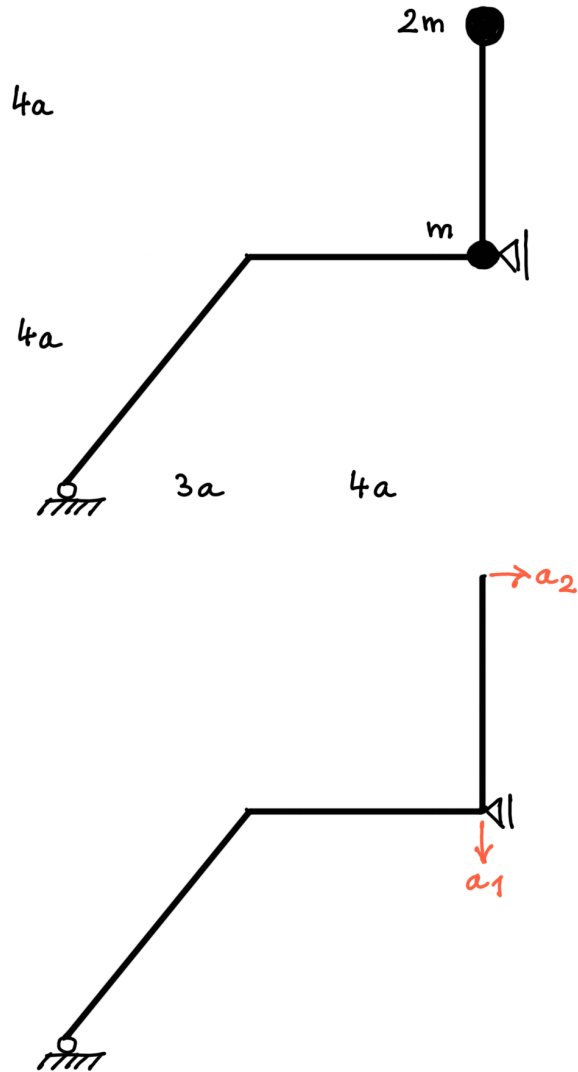


Test 2.3 2025/26 (MoS2 course)

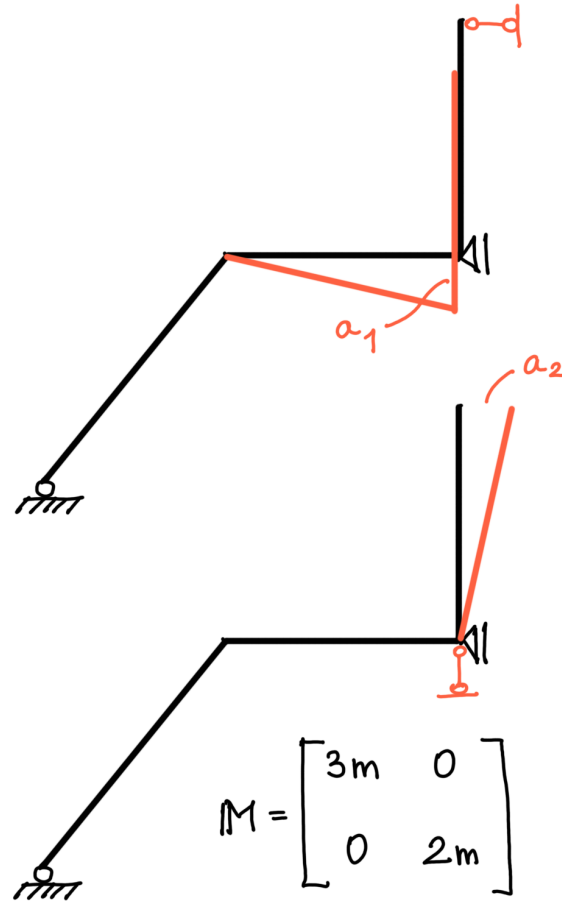
$EA = \infty$, EJ - const., $\mu = 0$

Compute the frequencies of natural vibrations in [Hz]



Lagrange coordinates

$$q(t) = a \cos(\omega t), \quad a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

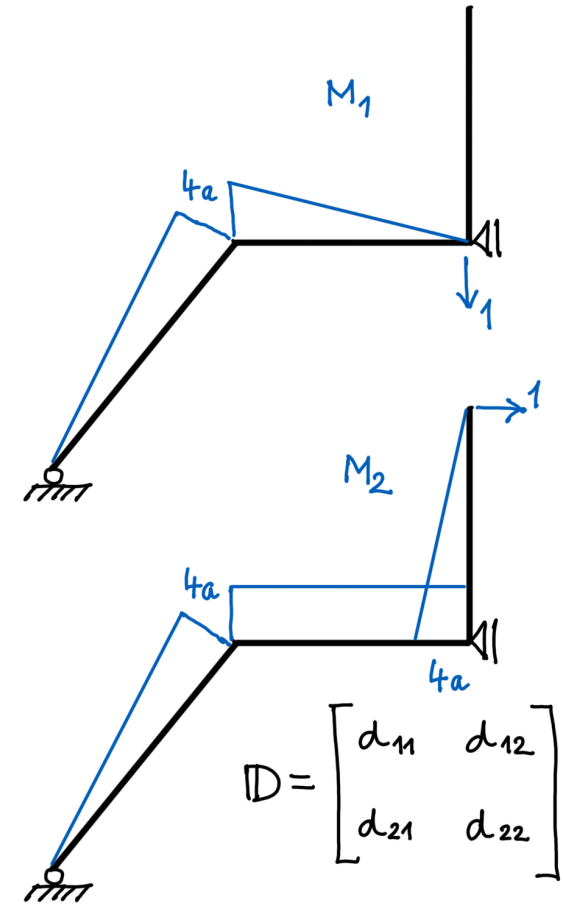


$$M = \begin{bmatrix} 3m & 0 \\ 0 & 2m \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(I - \omega^2 DM) a = 0$$

$$\det(I - \omega^2 DM) = 0 \rightarrow \omega_1, \omega_2 \rightarrow f_1, f_2$$



$$D = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

$$d_{11} = M_1 \square M_1 = 48 \frac{a^3}{EJ}$$

$$d_{12} = d_{21} = M_2 \square M_1 = \frac{176}{3} \frac{a^3}{EJ}$$

$$d_{22} = M_2 \square M_2 = 112 \frac{a^3}{EJ}$$

$$\mathbb{I} - \omega^2 \mathbb{D} \mathbb{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{\omega^2 m a^3}{EJ} \begin{bmatrix} 48 & \frac{176}{3} \\ \frac{176}{3} & 112 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 - 144x & -\frac{352}{3}x \\ -176x & 1 - 224x \end{bmatrix}$$

$$x = \omega^2 \frac{m a^3}{EJ}$$

$$\det(\mathbb{I} - \omega^2 \mathbb{D} \mathbb{M}) = 11605 x^2 - 368x + 1$$

$$\det(\mathbb{I} - \omega^2 \mathbb{D} \mathbb{M}) = 0 \rightarrow x_1 = 0,028 \rightarrow \omega_1 = \sqrt{x_1} \sqrt{\frac{EJ}{ma^3}} = 0,167 \sqrt{\frac{EJ}{ma^3}} \rightarrow f_1 = \frac{\omega_1}{2\pi} = 0,027 \sqrt{\frac{EJ}{ma^3}} \text{ [Hz]}$$

$$x_2 = 0,003 \rightarrow \omega_2 = \sqrt{x_2} \sqrt{\frac{EJ}{ma^3}} = 0,055 \sqrt{\frac{EJ}{ma^3}} \rightarrow f_2 = \frac{\omega_2}{2\pi} = 0,009 \sqrt{\frac{EJ}{ma^3}} \text{ [Hz]}$$